Measurement and simulation of rarefied Couette Poiseuille flow with variable cross section

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Measurement and simulation of rarefied Couette Poiseuille flow with variable cross section

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Clearance flows are the main loss mechanism in dry running positive displacement vacuum pumps. In order to calculate the operation of those pumps, a detailed knowledge of the clearance mass flow rates is crucial. The dimensions of such pumps and the large pressure range of the operating points require a wide range of gas rarefaction to be taken into account. The clearance flow can be described by a combined Couette Poiseuille flow due to the pressure gradient between two chambers and the rotation of the rotary pistons. These clearance flows are studied experimentally and theoretically in the present work. Therefore, a suitable experimental setup is described together with the requirements of sensors and the necessity of a low leakage. A theoretical approach is presented, and the results are compared to experimental investigations varying the pressure ratio and the circumferential speed of a clearance boundary in a wide range of the gas rarefaction.

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I. INTRODUCTION

Dry running vacuum pumps have become an important part of vacuum systems due to their oil-free working principle. The need for a clean vacuum, for instance in the semiconductor industry, has pushed the development of dry running positive displacement pumps, such as screw and roots pumps (see Ref. 1). Due to contactless operation of the pistons the sealing between the working chambers is realized by small clearances. Regarding Fig. 1, the effective mass flow rate of a screw machine can be calculated by the delivered mass flow rate \( \dot{m}_d \) of a working chamber driven by the rotation of the pistons and reduced by the clearance mass flow rate \( \dot{m}_c \). Clearances typically appear between both rotary pistons and the housing in such machines. Calculating the thermodynamical operation of such pumps, the knowledge of these clearance mass flow rates is essential, which represent the main loss mechanism. An exemplary clearance geometry is shown in Fig. 1 separating two working chambers of different pressure and temperature of the working fluid. The clearance can be described by a minimal height \( h_{\text{min}} \), a width \( w \), a length \( L \) and a radius \( R \) regarding the investigated geometry in the current work. Due to the rotation of the rotary pistons a wall velocity \( U \), to be defined in the direction of the shortest clearance length, appears.

Screw vacuum pumps are deployed as fore vacuum pumps and the suction pressure ranges from atmospheric state down to only a few Pascal, meaning that the pressure ranges in more than five decades. In addition the circumferential speed of the tip of the rotary piston can reach up to 80 m/s. In this context two basic flow problems can be identified in order to predict the mass flow rate in such clearances. The Couette flow which is driven by the movement of the rotary piston and the Poiseuille flow driven by the pressure gradient between two working chambers. In addition, the prediction of the mass flow rate is complicated by the fact that rarefaction effects of the clearance flow have to be taken into account due to the large pressure range. The degree of rarefaction is expressed by the Knudsen number, which can be described by the ratio of the mean free path of the molecules and a characteristic length (the minimal clearance height \( h_{\text{min}} \)). Typically all flow types appear in these machines, ranging from the continuum flow to the free molecular flow.

The rarefied gas flow in channels, tubes and clearances has been studied extensively in the field of microelectromechanical systems during the past decades. These systems often operate at atmospheric flow conditions with very small length scales (see Ref. 2). Thus, the range of Knudsen numbers which has to be dealt with is comparable to clearance flows in vacuum pumps. Regarding the literature various approaches modeling rarefied flows are present. While in the continuum flow regime normally the Navier-Stokes equations are used applying a no-slip boundary condition on the walls, this approach fails at higher Knudsen numbers. Therefore, in the so called slip flow regime a slip boundary condition at the walls can be introduced to improve the

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Fig. 1. Screw vacuum pump and housing clearance.
results of the Navier-Stokes equations. The application of a first order slip model fails to predict the Knudsen minimum, so that a second order model has to be introduced (see Ref. 2). If the Navier-Stokes equation fail at moderately high Knudsen numbers, one can calculate the rarefied flow on the basis of the Boltzmann equation, but the solution is complicated because of the collision term (see Refs. 4 and 5). A widely used method for the solution of the collision term is the Bhatnagar–Gross–Krook model (BGK-model) described in Ref. 6. The idea of the model is that due to the collision term the distribution function tends to equilibrium state. Often this is combined with a linearization of the Boltzmann equation. Cercignani uses this method to determine the volume flow rate between two parallel plates and compares it with experimental data (see Ref. 7). The method is suitable to predict the Knudsen minimum. In addition to the BGK-model different models for the collision term exist, like the S-model in order to represent the correct Prandtl number. Sharipov uses this model to calculate a Poiseuille and thermal creep flow through a long tube and parallel plates. A method for solving the Boltzmann equation statistically is provided by the direct simulation Monte Carlo (DSMC) method (see Ref. 4), which considers a discrete set of particles, each owning a position and velocity. Key feature of the method is the decoupling of molecular movement and intermolecular interactions. The method has been used widely in the context of microdevices, see Refs. 12–16.

As described earlier, two basic flow problems have to be handled for calculating the mass flow rate in clearances of vacuum pumps, the Couette flow and the Poiseuille flow. For both flow problems, various investigations can be found in the literature.

The linearized Boltzmann equation often is used to calculate the flow rate for a pressure driven flow between two parallel plates. Sharipov (see Ref. 20) determines the flow rate in a channel of finite width. In Ref. 21, Cercignani et al. calculate the flow rate for the Poiseuille flow with different accommodation coefficients of both plates. Kosuge and Takata propose a database for Poiseuille, thermal transpiration, and concentration-driven flow for binary gas mixtures.

The literature provides an extensive amount of experimental data for the validation of flow rates determined by theory concerning a Poiseuille flow. Harley et al. and Turner et al. investigate the friction factor in microchannels. Ewart et al. determine the mass flow rate in a microchannel using the pressure rise method in a single tank. Arkilic uses a two-tank approach for his measurements in order to minimize the effect of temperature fluctuation throughout the measurement (see Refs. 3 and 26). In Ref. 27, Varoutis et al. investigate the conductance of channels with different cross sections, and Veltzke and Hemadri consider a channel with a variable cross section. Concerning the rarefied Poiseuille flow, Veltzke also gives a very detailed overview of experimental works.

Sone (see Ref. 30) investigates the velocity profile and the shear stress of a plane Couette flow on the basis of the linearized Boltzmann equation. Li and Cercignani et al. use the linearized Boltzmann equation to investigate the flow rate of a plane Couette flow for different accommodation coefficients. Results for the plane Couette flow using the DSMC method can be found in Ref. 33, where the velocity profile is calculated for different Knudsen numbers. Further on, Bao et al. (see Ref. 34) calculate the velocity slip and the temperature jump for the plane Couette flow for different Knudsen numbers by help of the DSMC method and compare it with other methods.

Experimental investigations concerning the Couette flow can be found in the field of measurement of the tangential momentum accommodation coefficient. Here, a spinning rotor gauge is used in order to measure the friction momentum in dependence of the Knudsen number (see Refs. 35 and 36).

The investigation of a combined Couette Poiseuille flow can be found in the context of thin film slider bearings. Fukui and Kaneko developed a generalized Reynolds-type lubrication theory in order to calculate the load capacity of such slider bearings. This is done by the superposition of the Couette and Poiseuille mass flow rate, which are calculated independently. Fukui and Kaneko and also Cercignani et al. determine the Poiseuille mass flow rate for different tangential accommodation coefficients via the linearized Boltzmann equation. Fukui and Kaneko then extend the model by a thermal creep flow. DSMC calculations were carried out by Alexander et al. and their findings are compared to the theory of Fukui et al., which show a good agreement. Bahukudumbi and Beskok developed a phenomenological model to describe the flow in a slider bearing using adequate velocity slip models and compare their findings with the results of Fukui et al. and Alexander et al. Zahid carried out investigations of a combined Couette Poiseuille flow with an overall pressure ratio which differs from unity. This is done by the solution of the full Navier-Stokes equation with a slip boundary condition to calculate the pressure and velocity distribution for slight rarefied flows. In the context of vacuum pumps, Sharipov et al. uses the superposition of Couette and Poiseuille flow to calculate the operating characteristic of a Holweck-pump.

As shown, the literature provides various approaches for the calculation of a combined Couette Poiseuille flow. On the other hand, there is a lack of experimental data concerning a combined Couette Poiseuille flow. Therefore, the aim of the present paper is the introduction of an experimental setup to investigate a combined Couette Poiseuille flow. Further on, the experimental results are compared to simulation results using a theoretical approach.

II. THEORETICAL APPROACH

The mass flow rate \( \dot{m} \) of a long rectangular channel connecting two reservoirs with the same gas in the whole range of the gas rarefaction shall be determined. The channel may have a variable cross section aspect ratio \( h/w \) and a moving boundary. The channel width \( w \) is supposed to be constant, but the channel height \( h \) may vary continuously along the channel under the condition \( \max(h) < w \). The reservoirs are maintained at pressure \( p_{\text{in}} \) at the inlet and \( p_{\text{out}} \) at the outlet. The temperature \( T_{\text{in}} = T_{\text{out}} = T \) is assumed to be constant.
The results will be given in the dimensionless reduced flow rate

\[ G = \frac{L \cdot c_m}{h_{\text{min}} \cdot w \cdot p_m \cdot \dot{m}} \]  

(1)
as a function of the Knudsen number, using the model of hard sphere molecules

\[ Kn = \frac{\mu \cdot c_m \cdot \sqrt{\pi}}{2 \cdot h_{\text{min}} \cdot p_m} \]  

(2)

L is the length of the channel, \( h_{\text{min}} \) is the minimum channel height, \( \mu \) is the viscosity, and

\[ c_m = \sqrt{\frac{2 \cdot k \cdot T}{m}} \]  

(3)
is the most probable molecular speed with the molecular mass \( m \) and the Boltzmann constant \( k \).

A. Governing equations

In analogy to Refs. 43 and 44 we assume the channel to be long [i.e., \( \max(h) \ll L \)], so that end effects can be neglected and the local pressure gradient can be considered small in any cross section; thus

\[ \zeta_p = \frac{h}{p} \frac{dp}{dx}, \quad |\zeta_p| \ll 1 \]  

(4)

holds. Here, \( x \) is the longitudinal coordinate with its origin in the first reservoir, \( h = h(x) \) is the local channel height, and \( p = p(x) \) is the local pressure.

Due to the pressure gradient between the two reservoirs and the moving boundary, a superposition of the Couette and Poiseuille flow has to be taken into account. Thus, the mass flow rate in a cross section is calculated as

\[ \dot{m} = h \cdot w \cdot p \cdot \left( 2 \cdot G_C \cdot \frac{U_s}{c_m} - G_P \cdot \zeta_p \right), \]  

(5)

where \( U_s = U_s(x) \) is the local wall velocity of the moving boundary in the flow direction. In a similar way, this can be found in Refs. 39 and 43.

While the reduced flow rate of the plane Couette flow \( G_C \) can be reduced to the constant value \( G_C = 1/2 \) due to the symmetry of the Couette flow (see Ref. 39), the reduced flow rate of the Poiseuille flow \( G_P = G_P(\delta, h/w) \) depends on the local rarefaction parameter \( \delta \) and the local aspect ratio \( h/w \). The rarefaction parameter \( \delta \) is defined as

\[ \delta = \frac{h \cdot p}{\mu \cdot c_m}. \]  

(6)

This expression implies the model of hard sphere molecules according to the definition of the Knudsen number \( Kn \) (see Ref. 20).

Equation (5) is solved numerically using the following finite difference scheme in analogy to Refs. 43 and 44:

\[ p_{i+1} = p_i + \frac{\Delta x}{G_P(\delta_i, h_i/w) \cdot h_i} \cdot \left( \frac{p_i \cdot U_{s,i}}{c_m} - \frac{\dot{m} \cdot c_m}{h_i \cdot w} \right). \]  

(7)

Here, \( \Delta x = L/(N - 1) \) is the increment in the \( x \) direction, \( N \) is the number of nodes, and \( 0 \leq i \leq N - 1 \) is the current grid point with the pressure \( p_i \), wall velocity \( U_{s,i} \), rarefaction parameter \( \delta_i \), and height \( h_i \) of the \( i \)th grid point, with

\[ \delta_i = \frac{h_i \cdot p_i}{\mu \cdot c_m}. \]  

(8)

Equation 7 can be solved numerically with the boundary condition \( p(x=0) = p_{\text{in}} \), where the mass flow rate \( \dot{m} \) is found via a bisection method satisfying the second boundary condition \( p(x=L) = p_{\text{out}} \).

B. Determination of the reduced flow rate \( G_P \)

1. Free molecular regime (\( \delta = 0 \))

In the free molecular regime (\( \delta = 0 \)), the reduced flow rate \( G_P \) can be calculated analytically as

\[ G_P = \left[ \frac{w}{h} \ln \left( \frac{h}{w} + \sqrt{1 + \frac{h^2}{w^2}} \right) + \ln \left( \frac{w}{h} + \sqrt{1 + \frac{w^2}{h^2}} \right) \right] \frac{1}{\sqrt{\pi}} \]  

(9)

according to Refs. 45 and 46.

2. Hydrodynamic regime (\( \delta \geq 20 \))

In the hydrodynamic and slip regime [\( \delta \geq 20 \) (Ref. 1)], the reduced flow rate can be calculated as

\[ G_P = \frac{\delta}{6} \cdot H + \sigma_P \cdot S, \]  

(10)

where \( \sigma_P = 1.016 \) is the viscous slip coefficient which is calculated by Sharipov via the BGK-Model with diffuse gas-surface interaction (see Ref. 20). For \( \delta \to \infty \), Eq. (10) tends to the solution of the Stokes equation. The coefficients \( H \) and \( S \) for the aspect ratio of a rectangular channel read

\[ H = 1 - \frac{192}{\pi^2} \frac{h}{w} \sum_{n=1}^{\infty} \frac{\tanh(0.5\pi(2n+1)w/h)}{(1+2n)^3}, \]  

(11)

and

\[ S = \frac{4}{3} - \frac{256}{\pi^5} \frac{h}{w} \sum_{n=0}^{\infty} \frac{\tanh(0.5\pi(2n+1)w/h)}{(1+2n)^3} - \frac{32}{\pi^3} \left( 1 - \frac{h}{w} \right) \sum_{n=0}^{\infty} \frac{\tanh(0.5\pi(2n+1)w/h)}{(1+2n)^4}, \]  

(12)

and can be derived from Refs. 20, and 44–46. The values of Eqs. (11) and (12) are obtained with an accuracy of 0.01%.

3. Transitional flow regime (\( 0 < \delta < 20 \))

In the transitional flow regime, no analytical solution is known to obtain the reduced flow rate. Thus, numerical
values depending on the rarefaction parameter and the cross section aspect ratio \( G_p(\delta, h/w) \) are needed and can be found in the tabulated data of Ref. 20. Sharipov obtained these results by simulations using the BGK-Model and solving the linearized Boltzmann equation using a discrete velocity method. A diffuse gas–surface interaction model is deployed, and \( G_p \) values are calculated in a large range of gas rarefaction parameter \( 0.001 \leq \delta \leq 20 \) and cross section aspect ratio \( 0.01 \leq h/w \leq 1 \). The analytical values of the free molecular regime \( \delta = 0 \) and the solution for infinite plates \( h/w = 0 \) are included. Detailed explanations concerning the applied model and the boundary conditions are given in Ref. 47.

In order to create a continuous course for the reduced flow rate \( G_p(\delta, h/w) \) from the discrete tabulated values, a bilinear interpolation method is used, which can be found in Chap. 3 of Ref. 48.
III. EXPERIMENTAL SETUP

A. Design of the experimental setup with moving boundary

Figures 2 and 3 depict the design of the clearance model, which can be used for the investigation of a combined Couette Poiseuille flow. The clearance (1) is formed by a rotary shaft (2) with a diameter of 150 mm on one side and by a plane contour (3) on the other side and has a width \( w \) of 270 mm. The inlet (4) and the outlet (5) of the clearance are formed by two volumes arranged in the housing (6), which have a height \( h(x = 0) \) and \( h(x = L) \) of 36 mm, and the inlet and the outlet pressure can be varied. The length \( L \) of the clearance is 127 mm. It is possible to adjust the minimal clearance height \( h_{\text{min}} \) in the range of 0–1 mm by displacing the plate (7) from the outside, which is connected to the plane contour (3). Two distance sensors (8) are used to measure the precise minimal clearance height at two locations of the clearance. Due to very low mass flow rates, which have to be measured, the major priority is to achieve a small overall leakage of the clearance model, which will be discussed in Sec. III D. Wherever a sealing is required, O-ring sealings are used throughout the whole design. For the drivetrain, a magnetic clutch (9) is used to avoid any dynamic sealings with a potential of high leakages. The outer rotor of the magnetic clutch (which is not shown in Fig. 2) is connected to an electric drive, which can be operated at variable speed. In addition to external leakages, which can be minimized by using O-ring sealings, internal clearances exist due to the rotatable shaft (2). A housing clearance (10), which separates the housing and the shaft and connects inlet and outlet, is present. Further on, two front clearances (11) exist, which separate the frontal area of the shaft and the bearing caps. These clearances connect the inlet and outlet of the clearance with the spacing of the bearings. As it will be described later, the measured clearance mass flow rate \( \dot{m}_{\text{cl}} \) has to be corrected by the mass flow rate of the housing clearance \( \dot{m}_{\text{h}} \). The front clearances act as a leakage and have therefore to be minimized. For this, two connectors (12) are located at the bearing spaces in order to evacuate these volumes. Additionally, to minimize leakages from the volume at the back of the plane contour (3) a third connector (13) is installed to evacuate this volume. Thus, a small overall leakage can be achieved.

In general, two different methods of mass flow rate measurement are used, the single tank method and the use of thermal mass flow sensors (MFS), which will be described in detail in Sec. III B. A complete description of different methods can be found in Refs. 28 and 49. Figure 4 shows the setup of the test rig, the assembled position of the clearance model, and the required equipment for the application of both methods of measurement. The air is sucked in at atmospheric state and passes one of overall six mass flow sensors (MFS1-6). To choose the desired measuring range, six magnetic valves (MV1-6) placed in parallel are available. Before the gas passes the throttle (Th) to adjust a specific mass flow rate, the inlet temperature \( T_{\text{in}} \) of the gas is measured. At the inlet and the outlet of the clearance model, the pressure \( p_{\text{in}} \) and \( p_{\text{out}} \) are measured. A roots and a screw vacuum pump are available in order to convey the gas. Using the single tank method to measure the mass flow rate, a recipient (R1) is available and can be switched on in addition by valve (V1). The recipient increases the volume at the inlet of the clearance in order to adjust a constant inlet pressure throughout the measurement. A second recipient (R2) is used to measure the pressure rise when the valve (V2) is closed rapidly. If the mass flow sensors are used recipient (R1) has to be disconnected by valve (V1) in order to reach a steady state of the flow faster and to omit higher leakages caused by the recipient (R1). The used sensors and its corresponding resolution and accuracy are listed in Table I.

B. Method of measurement

Using the single tank method in order to measure a small mass flow rate, recipient (R2) is placed at the outlet of the clearance model and a low leakage of the outlet volume has to be ensured. To conduct a measurement, a desired inlet pressure is adjusted at the inlet of the model using the throttle (Th), while the valve (V2) is open and the vacuum pumps convey the gas. In order to accomplish a constant inlet pressure throughout the measurement valve (V1) is
opened so that recipient (R1) is connected. Valve (V2) is then closed rapidly, and the recipient (R2) is flooded due to the clearance mass flow rate of the model. The change of the outlet pressure is divided into the clearance mass flow rate and the housing. This causes a mass flow rate that has to be determined for various operating points. It can be seen that the mass flow rate is a function of pressure and temperature gradient. The fact that measurements are conducted at a very low pressure \( p_{\text{out}} \) and the temperature gradient is small allows to determine the mass flow rate only using the pressure gradient.

The measurement of a higher mass flow rate is done using the mass flow sensors, which are based on a calorimetric principle. To ensure a low leakage of the inlet volume, which is necessary for this method of measurement, valve (V1) is closed so that recipient (R1) is disconnected. The desired outlet pressure \( p_{\text{out}} \) is adjusted by the speed of the vacuum pumps.

As depicted in Fig. 3, the design of the clearance model requires a housing clearance in order to separate the shaft and the housing. This causes a mass flow rate \( \dot{m}_b \), which has to be considered interpreting the experimental results. Neglecting external leakages, the measured mass flow rate \( \dot{m}_m \) is divided into the clearance mass flow rate \( \dot{m}_{c,l} \) and the housing mass flow rate \( \dot{m}_h \)

\[
\dot{m}_{c,l} = \dot{m}_m + \dot{m}_h.
\]  

The flow in the housing clearance can be regarded as a combined Couette Poiseuille flow analogously. The length of the clearance is 343 mm with a constant height of 0.12 mm. The geometry suggests that the flow in the housing clearance is highly influenced by only the Couette flow and the Poiseuille flow can be neglected. Following the described method in Sec. II according to Eq. (7), the housing clearance mass flow rate was calculated for various operating points. It was found that the Poiseuille flow in the housing clearance is very small due to the great length. Thus, the measured mass flow rate \( \dot{m}_m \) is corrected by the mass flow rate of a pure Couette flow in the housing clearance to calculate the clearance mass flow rate \( \dot{m}_{c,l} \). \( A_h \) is the area of the housing clearance, and \( U \) the circumferential speed of the shaft. For a static boundary (\( U = 0 \)), the clearance mass flow rate \( \dot{m}_{c,l} \) equals the measured mass flow rate \( \dot{m}_m \)

\[
\dot{m}_{c,l} = \dot{m}_m + \frac{p}{RT} A_h U^2.
\]  

Equation (15) has to be adjusted depending on the direction of circumferential speed. Following the notation in Fig. 3 positive and negative direction of rotation can be considered for the circumferential speed. Pressure \( p \) has to be replaced by the outlet pressure \( p_{\text{out}} \) regarding a positive circumferential speed; in contrast, the inlet pressure \( p_{\text{in}} \) has to be used regarding a negative circumferential speed. The temperature \( T \) is assumed to be identical to the inlet temperature \( T_{\text{in}} \) in both cases.

### C. Uncertainty of measurement

The described methods of measurement and correction by the housing clearance to obtain the clearance mass flow rate \( \dot{m}_{c,l} \) yield uncertainties. In general, the uncertainty \( \Delta Y \) of a quantity \( Y \) depending on noncorrelating quantities \( X_i \) is obtained by

\[
\Delta Y = \left( \sum_i \left( \frac{\partial Y}{\partial X_i} \Delta X_i \right)^2 \right)^{1/2},
\]  

with \( \Delta X_i \) being the uncertainties of \( X_i \); see Ref. 50.

Regarding Eq. (14), the uncertainty of the clearance mass flow rate reads

\[
\Delta \dot{m}_{c,l} = \sqrt{(\Delta \dot{m}_m)^2 + (\Delta \dot{m}_h)^2},
\]  

where \( \Delta \dot{m}_h \) and \( \Delta \dot{m}_m \) are the absolute uncertainties of the housing clearance mass flow rate and the measured mass flow rate. Estimating the uncertainty of the measured mass flow rate, it has to be distinguished between the methods of measurement—the single tank method and the usage of the mass flow sensors

\[
\frac{\Delta \dot{m}_m}{\dot{m}_m} = \sqrt{\left( \frac{\Delta V_{\text{out}}}{V_{\text{out}}} \right)^2 + \left( \frac{\Delta T_{\text{out}}}{T_{\text{out}}} \right)^2 + \left( \frac{\Delta p_{\text{out}}}{p_{\text{out}}} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_h}{\dot{m}_h} = \sqrt{\left( \frac{\Delta p}{p} \right)^2 + \left( \frac{\Delta T}{T} \right)^2 + \left( \frac{\Delta h_f}{h_f} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \frac{\Delta \dot{m}_m}{\dot{m}_m} + \frac{\Delta \dot{m}_h}{\dot{m}_h},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \frac{\Delta \dot{m}_m}{\dot{m}_m} + \frac{\Delta \dot{m}_h}{\dot{m}_h} + \frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}}.
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \sqrt{\left( \frac{\Delta p}{p} \right)^2 + \left( \frac{\Delta T}{T} \right)^2 + \left( \frac{\Delta h_f}{h_f} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \sqrt{\left( \frac{\Delta V_{\text{out}}}{V_{\text{out}}} \right)^2 + \left( \frac{\Delta T_{\text{out}}}{T_{\text{out}}} \right)^2 + \left( \frac{\Delta p_{\text{out}}}{p_{\text{out}}} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \sqrt{\left( \frac{\Delta V_{\text{out}}}{V_{\text{out}}} \right)^2 + \left( \frac{\Delta T_{\text{out}}}{T_{\text{out}}} \right)^2 + \left( \frac{\Delta p_{\text{out}}}{p_{\text{out}}} \right)^2 + \left( \frac{\Delta \dot{m}_m}{\dot{m}_m} \right)^2 + \left( \frac{\Delta \dot{m}_h}{\dot{m}_h} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \sqrt{\left( \frac{\Delta V_{\text{out}}}{V_{\text{out}}} \right)^2 + \left( \frac{\Delta T_{\text{out}}}{T_{\text{out}}} \right)^2 + \left( \frac{\Delta p_{\text{out}}}{p_{\text{out}}} \right)^2 + \left( \frac{\Delta \dot{m}_m}{\dot{m}_m} \right)^2 + \left( \frac{\Delta \dot{m}_h}{\dot{m}_h} \right)^2},
\]  

\[
\frac{\Delta \dot{m}_{c,l}}{\dot{m}_{c,l}} = \sqrt{\left( \frac{\Delta V_{\text{out}}}{V_{\text{out}}} \right)^2 + \left( \frac{\Delta T_{\text{out}}}{T_{\text{out}}} \right)^2 + \left( \frac{\Delta p_{\text{out}}}{p_{\text{out}}} \right)^2 + \left( \frac{\Delta \dot{m}_m}{\dot{m}_m} \right)^2 + \left( \frac{\Delta \dot{m}_h}{\dot{m}_h} \right)^2}}.
\]  

### Table I. Technical data of sensors in use.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>Full scale (FS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow sensors (MFS):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKS 258C, MKS 558A</td>
<td>0.1% FS</td>
<td>±1% FS</td>
<td>0.1, 1, 10, 50, 100, 200 (NL/min)</td>
</tr>
<tr>
<td>Pressure sensors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKS Baratron 627B</td>
<td>10^{-3}% FS</td>
<td>±0.12%–0.15% of reading</td>
<td>0.1, 1, 10, 100, 1000 (mbar)</td>
</tr>
<tr>
<td>Temperature sensors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermocouple type T</td>
<td>—</td>
<td>±0.5 K</td>
<td>−185 to 400 (°C)</td>
</tr>
<tr>
<td>Distance sensors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capaNCDT 6110</td>
<td>0.01% FS</td>
<td>±0.15% FS</td>
<td>1 (mm)</td>
</tr>
</tbody>
</table>
The uncertainties concerning the pressure, temperature, and mass flow rate measurement are given in Table I. The outlet temperature $T_{out}$ is assumed to be identical to the inlet temperature $T_{in}$. Due to the setup of the outlet volume $V_{out}$ with several vacuum components and fittings, the relative uncertainty $\Delta V_{out}/V_{out}$ in the determination of the volume is assumed to be $\pm 2\%$. The height of the housing clearance $h_h$ is quantified by using a feeler gauge with an estimated absolute uncertainty $\Delta h_h = \pm 0.01 \text{ mm}$. Since the width of the housing clearance is large ($w = 270 \text{ mm}$), this uncertainty is neglected. Also, the determination of the circumferential velocity $U$ is disregarded. The time derivative of the outlet pressure $p_{out} = \frac{dp_{out}}{dt}$ is taken from the gradient of a linear regression in a time interval of $\Delta t = 0.15 \text{ s}$. To estimate the uncertainty $\Delta p_{out}$, the gradient of a linear regression of different pressure sensors is compared. The maximum deviation of the evaluated sensors is smaller than $2\%$; thus, an uncertainty $\Delta p_{out}/p_{out} = \pm 2 \%$ is estimated.

D. Leakage of the setup

Measuring the mass flow rate of a certain clearance is very challenging in the presence of additional clearances. While the mass flow rate of the housing clearance can be predicted as being discussed in Sec. III B, there is a lack of knowledge concerning the front clearances. Therefore, the front clearance flows are part of the integral leakage mass flow rate

$$m_L = \frac{V}{RT} \frac{dp}{dt},$$

being measured for various pressure decades via the pressure rise method with $V$ being the volume of the whole system. To compare the leakage mass flow rate with the results in Sec. IV, the reduced leakage flow rate $G_L$ is used according to Eq. (1), using the clearance dimensions. Figure 5 shows the reduced leakage flow rate $G_L$ as a function of the Knudsen number related to the minimum height $h_{min} = 0.3 \text{ mm}$ of the clearance model.

Due to the evacuation of the spacing of the bearings, a pressure dependence of the leakage occurs which leads to a change of the sign of the leakage mass flow rate at a system pressure of $25 \text{ Pa}$. When the system pressure is lower, an inflow occurs in the front clearances, which has the same sign as the leakage through sealings from the atmospheric state. At a higher system pressure, the leakage mass flow rate gets a negative sign, thus an outflow occurs due to the front clearances. It must be noted that the determination of the leakage mass flow rate is carried out with a boundary velocity of $U = 0 \text{ m/s}$ and a pressure ratio of $\Pi = p_{out}/p_{out} = 1$, which is not the case for measurements shown in Sec. IV. Because the moving boundary is directed transversely to the front clearances, the assumption is made that the wall velocity does not increase the leakage mass flow rate. Nevertheless, for most measuring points, the resulting leakage mass flow rate is small compared to the measured clearance mass flow rate.

IV. RESULTS

In order to evaluate the method of measurement a comparison of the single tank method and the mass flow sensors is conducted. Further on, values of the corrected reduced flow rate are then compared with simulation results. To determine the influence of a moving boundary in rarefied gases, a variation of the wall velocity $U$ is carried out for a constant pressure ratio $\Pi = p_{out}/p_{out}$ and a minimal clearance height $h_{min}$. In a second step, the pressure ratio $\Pi$ is varied for a constant boundary velocity $U$ and a minimal clearance height $h_{min}$. The analysis of the experimental data is conducted according to Sec. III B, and the results are shown together with the experimental error described in Sec. III C. Simulation results are shown according to the theoretical approach described in Sec. II using an uniform temperature of $T = T_{in} = 293 \text{ K}$.

A. Comparison of methods of measurement

Figure 6 shows experimental data of the reduced flow rate as a function of the Knudsen number for a pressure ratio of $\Pi = 8$, a circumferential speed of $U = -20 \text{ m/s}$, and a minimal clearance height of $h_{min} = 0.3 \text{ mm}$. The filled marks represent the data of the measured mass flow rate $m_m$. The corrected data of the clearance mass flow rate $m_{cl}$ (empty marks) according to Eq. (15) are shown together with the experimental error described in Sec. III C. For Knudsen numbers in the range of $0.02 < Kn < 0.1$, data for the reduced flow rate using both methods are present. It can be seen that both results are in very good agreement. Regarding the experimental data of the reduced flow rate using the corrected mass flow rate $m_{cl}$, a very good agreement with the results according to the theoretical approach described in Sec. II can be seen. Thus, the method of measurement seems to be suitable, and the following results for the reduced flow rates $G$ are calculated using the clearance mass flow rate $m_{cl}$.

B. Variation of wall velocity $U$

In a first step, the circumferential speed shall be varied in the range from $-40$ up to $40 \text{ m/s}$ for a pressure ratio $\Pi = 8$ and a minimal clearance height $h_{min} = 0.3 \text{ mm}$ (see...
Fig. 7). According to the classification of flow regimes in Ref. 51, experimental investigations are carried out for continuum ($Kn < 0.01$), slip ($0.01 < Kn < 0.1$), and transitional ($0.1 < Kn < 10$) flow regimes. Regarding the experimental results of a stationary boundary ($U = 0$ m/s), typical features of the reduced flow rate as a function of the Knudsen number can be observed. For very small Knudsen numbers, the reduced flow rate converges to a constant value, which can be explained by a choked flow in the clearance. With higher Knudsen numbers, the influence of frictional losses rises, resulting in a linear decrease in the reduced flow rate. In the slip flow regime, a velocity slip of the flow at the walls occurs, which reduces the effect of frictional losses. The transitional flow is denoted by the Knudsen minimum, located at a Knudsen number of $Kn \approx 0.8$. For higher Knudsen numbers, the reduced flow rate increases. Regarding the experimental results of a positive circumferential speed, an increase in the reduced flow rate for the transitional flow regime can be observed clearly. This effect is smaller in the slip and continuum flow regime. Additionally, the Knudsen minimum is shifted toward lower Knudsen numbers regarding a positive circumferential speed. Due to a limited suction speed of the vacuum pumps, the investigation of higher Knudsen numbers with a
circumferential speed of $U = 20 \text{ m/s}$ and $U = 40 \text{ m/s}$ is not possible. A negative circumferential speed decreases the reduced flow rate, and the Knudsen minimum is shifted to higher Knudsen numbers. Again, the effect of a moving boundary is higher in the transitional flow regime than in the slip and continuum flow regimes. For a circumferential speed of $U = -40 \text{ m/s}$, a negative dimensionless flow rate can be observed for $Kn > 0.06$. The experimental results show clearly that the influence of the Couette flow rises with higher Knudsen numbers and the greatest impact can be seen in the transition flow regime. In contrast, the continuum flow range is dominated by the Poiseuille flow.

The comparison of experimental and simulation data reveals a very good agreement in a large range of Knudsen numbers and circumferential speeds. The theoretical approach is able to depict the experimental data in the transitional and slip flow regimes. Minor deviations can be found for a circumferential speed of $U = -20 \text{ m/s}$, a negative dimensionless flow rate can be observed for $Kn > 0.06$. The experimental results show clearly that the influence of the Couette flow rises with higher Knudsen numbers and the greatest impact can be seen in the transition flow regime. In contrast, the continuum flow range is dominated by the Poiseuille flow.

In a second step, a variation of the pressure ratio shall be regarded, in a range from $\Pi = 1$ up to $\Pi = 6$ for a constant circumferential speed $U = 20 \text{ m/s}$ and a clearance height $h_{\text{min}} = 0.3 \text{ mm}$ (see Fig. 8). Again, experimental investigations are carried out in the continuum, slip, and transitional flow regimes. Regarding a high pressure ratio $\Pi = 6$, similar results can be observed, which are already described in Fig. 7. For smaller pressure ratios, a decrease in the reduced flow rate appears, especially in the continuum and slip flow regimes. In the case of identical inlet and outlet pressures ($\Pi = 1$), the reduced flow rate does not depend on the Knudsen number in the slip and continuum flow regimes.

The simulation results using the theoretical approach yield a very good accordance to the measurements. Again, the transitional and slip flow regimes are captured with high accuracy by the theoretical approach. Deviations between simulation and experiment can be found in the continuum flow regime and a pressure ratio $\Pi = 6$, which can be explained by a choked flow again. This deviation diminishes for smaller pressure ratios, because the consideration of an incompressible flow gets more suitable. Minor deviations in the slip and continuum flow regimes for a pressure ratio $\Pi = 1$ are due to the difficulty adjusting a steady state of the flow using the vacuum pumps at a high inlet pressure.

V. CONCLUSION

In the present paper, the combined Couette Poiseuille flow of a rarefied gas is investigated experimentally and the results are compared with simulations using a theoretical approach. The variation of the pressure ratio and the circumferential speed of the clearance boundary show that the theoretical approach is very suitable to describe the flow in such
clearances. An excellent agreement can be found in the transitional and slip flow regimes. In the continuum flow regime, the theoretical approach is also able to depict the flow for small pressure ratios; however, it fails to predict the flow of high pressure ratios due to a choked flow in the clearance. It can be shown that the flow in such clearances is highly influenced by the Couette flow for high Knudsen numbers, whereas the flow is mostly dominated by the Poiseuille flow for small Knudsen numbers.

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