Geometrical abstraction of screw compressors for thermodynamic optimization

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Abstract: The construction and development of different rotor profiles is an important area in connection with the development of screw compressors for specific applications. Geometrical performance figures (using criteria to describe interdependencies of geometrical parameters for screw compressors) for profile optimization are used in order to achieve specific improvements in performance. During this process, rotor profiles and spatial parameters are the main factors. Compared to data derived from the front section of rotor profiles, these figures which also take spatial parameters into account provide a better evaluation of gap conditions and operating efficiency of the compressors under examination.

Keywords: screw compressor performance, profile optimization, new design concepts

1 INTRODUCTION

The operational behaviour of a screw compressor can be examined experimentally but also by means of comprehensive simulation procedures [1,2]. The complexity of the simulation, the need for trial runs, and the need to achieve the closest possible parallels to a real compressor do lead to valid results, but the process is very time-consuming. Computer-aided optimization by comprehensive simulation is therefore undesirable, despite leading to valid results. With a view to finding a compromise between precision and calculation time, evaluation and analysis by means of abstracted geometrical performance figures is a reasonable approach. The main feature of this approach is intended to be a reduction in development time by using geometrical codes to characterize the thermodynamic performance behaviour of a screw compressor.

Within the field for developing geometrical performance figures, there have been attempts to evaluate the gap conditions for various front-section geometries [3]. This approach has proved useful in spite of the two-dimensional (2D) viewpoint, and demonstrates that direct profile modification taking into account application-orientated requirements seems to be effective. However, this approach does not provide direct comparability of 3D rotor geometries, as a pure profile abstraction inevitably ignores spatial geometrical parameters. These exercise considerable influence on gap conditions, and thus on thermodynamic processes in the compressor.

The aim of this study is to link geometrical parameters with their effects in terms of performance. The interrelationships will be integrated in geometrical performance codes, which can then be used to reduce internal leakages and, to some extent, improve the energy conversion efficiency of screw compressors. Ascertaining general trends for geometrical optimization will be carried out via a purely geometrical evaluation of the compressors, independent of any actual operating situation. In order to compare the geometries, it is only necessary to decide which mechanical and operational parameters it is advisable to keep constant. It will be necessary to ascertain whether simple geometrical data are capable of replacing extensive measurements and simulations in a first assessment of the energy conversion efficiency of differing compressor designs, for example, within the framework of a computer-aided optimization process.

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2 INTERRELATIONSHIPS BETWEEN GEOMETRICAL PARAMETERS

The selection of the geometrical parameters of a compressor, such as rotor diameter, length, rotor wrap angle, and compression ratio, influences gap interaction and the resulting degree of energy conversion. The settings for the casing, front, and profile intermesh are responsible for the internal leakage characteristics of the compressor, and are mainly responsible for gap flow losses (Fig. 1). If gap flow losses are increased, efficiency suffers. The types of gaps have different influences on the energy conversion efficiency of a screw compressor. A general view of gap priorities with regard to the relative rates of mass flow at the gaps—respect to the current pressure situation—and the respective gap lengths is provided by [1, 4] the following.

1. Profile intermesh gap (between male and female rotor).
2. Casing gap at (depending on number of teeth):
   (a) male rotor;
   (b) female rotor (larger number of teeth than the male rotor).
4. Front gap at:
   (a) high-pressure side;
   (b) low-pressure side.

The order above refers basically to the overpressure area of a screw compressor. For a machine of this type used in mechanical compression applications, this order of priorities is confirmed by Kauder and Janicki [4]. Results for gap priorities in expansion applications are not available at present. The energy conversion assessment is carried out via mechanical efficiency as the principal performance figure. A further important item is the volumetric efficiency for rotational displacement machines.

The framework of the geometrical assessment does not include factors which have no direct influence on the rotor geometry. The level of volumetric efficiency thus represents the influence of the gap mass flows. Volumetric efficiency is influenced by geometrical parameters such as the way rotor tooth counts are paired, length of the rotor, wrap angle, length-diameter ratio, and the settings of the gap heights.

In addition, profile design plays a major role, because it is the form of the profile which influences the main types of gap [3, 4]. Based on a reference compressor, the influence of the profile form on the volumetric efficiency of the compressor is variable. Every change in the profile form directly influences the gap configuration and the order of priority among the gaps, which in turn has a direct influence on leakages in the compressor. The fact that changing the profile directly affects the size of the machine (i.e. the maximum delivery volume) should also be taken into account.
account. The smaller the delivery volume, the larger the effect of the gaps, as the increase in length is linear, while the delivery volume increases at the power of three related to the size of the compressor.

### 3 GAPS: A GEOMETRICAL EXAMINATION

Within the framework of profile development for dry-running rotational displacement machines, geometrical performance figures are helpful in carrying out a comparative assessment of different profiles. This can be done using either thermodynamic or mechanical flow values. Ignoring rotor length and wrap angles of the rotor under examination, rotor profiles with 2D performance figures (e.g. with 2D gap lengths in relation to the scoop surface), have so far been physically characterized. However, the part played by the spatial gap lengths is not taken into account. It seems desirable to transfer the geometrical compressor parameters (wrap angle and rotor length) to the resulting gap situation. Gap priority now depends on the settings of the gap height and the profile contour itself. Consequently, various approaches to a description of the gap conditions should be set up and validated within the framework of a computer-aided profile optimization procedure.

#### 3.1 Geometrical gap situation

An assessment of 3D gap interrelationships can be effectively represented by means of a rotor diagram (Fig. 2). The rotor position is determined by the leading chamber, where the volume has just reached zero. As the influence of the front gap on the low-pressure side is small, this is not taken into account in the performance figures. A comparison of different wrap angles shows that as the angles increase, the numbers and also the total lengths of the gaps increase. Where there is an identical pressure ratio, greater wrap angles will result in a more constant pressure gradient, which should result in higher volumetric efficiency. This assessment only applies if there is a constant theoretical mass flow in comparable compressors.

Gap conditions can be derived from the representation of the rotors, as a single gap alteration (e.g. in the length or the height), changes its priority and its influence on volumetric efficiency. Based on the reference compressor (compressor with subscript 11), simply combining values for the surfaces of the respective gaps produces an approximation of the performance relation \( \Pi_1 \), because it is assumed that the volumetric efficiency will fall in proportion to the gap area (equation (1)). The gap area \( A_{\text{Gap}} \) is arrived at by adding the gap lengths, multiplied in each case by the gap height. Comparability of different compressors can be

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**Fig. 2** Rotor intermesh diagram demonstrating the gap analysis for a specified rotor position. Above: small wrap angle, below: large wrap angle. CG, casing gap; BH, blow hole; IMC, intermesh clearance; HP, high pressure
achieved with reference to the delivery volume (i.e. depending on the number of teeth)

\[
\frac{\Pi_1}{(\Pi_1)_{11}} = \frac{A_{\text{Gap}}}{(A_{\text{Gap}})_{11}} \frac{(V_{\text{max}ZMR})_{11}}{V_{\text{max}ZMR}}
\]

with

\[
A_{\text{Gap}} = \sum (h_{MC} l_{MC}) + \sum A_{\text{SH}}
+ \left( \sum (h_{CG} l_{CG}) + \sum (h_{FG} l_{FG}) \right)_{\text{MR}}
+ \left( \sum (h_{CG} l_{CG}) + \sum (h_{FG} l_{FG}) \right)_{\text{FR}}
\]

This approach only provides a rough estimate of the volumetric efficiency in an assessment of profile characteristics with different wrap angles. This is because gap area does not necessarily change along with the wrap angle, whereas volumetric efficiency develops in an approximately anti-proportional way.

Gap influences tend to reduce as wrap angles increase, with the total number of gaps increasing. We can conclude from this that the gap count tends to develop in an approximately anti-proportional way compared with the gap area, so the equation can be extended as follows

\[
\frac{\Pi_2}{(\Pi_2)_{11}} = \frac{\Pi_1}{(\Pi_1)_{11}} \frac{(i_{\text{Gap}})_{11}}{l_{\text{Gap}}}
\]

with

\[
i_{\text{Gap}} = \sum (l_{\text{AC}} + l_{\text{BH}} + (l_{CG} + l_{FG})_{\text{MR}} + (l_{CG} + l_{FG})_{\text{FR}})
\]

The performance code \( \Pi_2 \) thus represents the relationship between a moderate gap area for all gaps of the compressor and the theoretical delivery volume. The number of gaps \( i_{\text{Gap}} \) is arrived at by adding together the total gaps for the machine. The total gap count broadly corresponds with the weightings of the gaps in a screw compressor. As the number of individual gap types does not change at the same rate (e.g. the wrap angle), it is desirable to carry out an assessment of the individual gap types.

### 3.2 Assessment of gap types

The previous calculations do not directly cater for gap areas dependent on wrap angles and rotor length. This means that the actual significance of the different gaps is not taken into account. The performance figures will be augmented by internal and external weighting factors for the respective gap areas (3)

\[
\frac{\Pi_3}{(\Pi_3)_{11}} = \frac{\sum (a_{\text{Gap},i}) \sum (a_{\text{Gap},i} A_{\text{Gap},\text{Type},i})_{11}}{\sum (a_{\text{Gap},i} A_{\text{Gap},\text{Type},i})_{11}} \frac{(V_{\text{max}ZMR})_{11}}{V_{\text{max}ZMR}}
\]

with

\[
a_{\text{Gap},i} = \frac{\sum A_{\text{Gap},\text{Type},i} i_{\text{Gap}}}{A_{\text{Gap}}} \text{ and } a_{\text{Gap},i} = \frac{A_{\text{Gap},\text{Type},i}}{\sum A_{\text{Gap},\text{Type},i}}
\]

Internal weighting factors are revealed by examining a single gap, with changes in the machine parameters also leading to changes in the number of gaps, and also in the total area of the gap under examination. The factor \( a_{\text{Gap},i} \) relates the specific area of a gap type to the total area of a gap type. At constant pressure, gaps with higher values have a positive effect on performance. External weighting factors for a particular gap type result from the machine gaps, evaluated via the gap area and count. The factor \( a_{\text{Gap},i} \) therefore represents the relationship between the mean area of each gap of a particular type and the mean area of all gaps. This performance code therefore combines all important gap-geometrical and variable values, which vary according to the profile form and intermesh characteristics. As by the creation of these codes the area of a gap type is entered in quadratic form, the surface component of the gap as a whole only in linear form, there is an extreme lack of proportion between the areas of the gap types, which results in an unsatisfactory representation of gap priorities.

### 3.3 Evaluation of a single-chamber examination

The working chamber, which is mainly responsible for the compression process, exercises a decisive influence on volumetric efficiency. With an increase in wrap angle, the total area of the gap increases, but the gap area of the process chamber is further reduced. Consequently, an examination of the high-pressure (HP) chamber in the previously defined rotor position can help to provide further gap performance values.

The generation of these values, referred to as \( \Pi_{1,\text{OCM}} \) and \( \Pi_{2,\text{OCM}} \), is carried out in the same way as \( \Pi_1 \) and \( \Pi_2 \). Code \( \Pi_{1,\text{OCM}} \) evaluates only the gap surfaces of the HP chamber, and does not take into account variations in rotor length with suitably modified wrap ratios (4)

\[
\frac{\Pi_{1,\text{OCM}}}{(\Pi_{1,\text{OCM}})_{11}} = \frac{A_{\text{Gap},1}}{(A_{\text{Gap},1})_{11}} \frac{(V_{\text{max}ZMR})_{11}}{V_{\text{max}ZMR}}
\]

with

\[
A_{\text{Gap},1} = A_{\text{MC},1} + A_{\text{BH},1} + (A_{\text{CG},1} + A_{\text{FG},1})_{\text{MR}}
+ (A_{\text{CG},1} + A_{\text{FG},1})_{\text{FR}}
\]

The gap area \( A_{\text{Gap},1} \) results from combining the individual gaps of the HP chamber, but this does not allow gap priorities to be ascertained. This influence is included via code \( \Pi_{2,\text{OCM}} \), with a weighting factor produced by the relationship between the HP side of the gap and the total area of the gap (equation (5)). With constant wrap angles, increasing the rotor length
inevitably leads to a more uniform pressure distribution throughout the machine, as there are more chambers between the high- and low-pressure sides.

\[
\Pi_3,0,CM = \frac{\sum (a_{\text{Gap},i}A_{\text{Gap,Type},i})}{\sum (\sum (a_{\text{Gap},i}A_{\text{Gap,Type},i}))} \frac{V_{\text{max},ZMR,11}}{V_{\text{max},5,ST}}
\]

with

\[a_{\text{Gap},i} = \frac{A_{\text{Gap,Type},i}}{\sum A_{\text{Gap,Type}}}\]

The weighting factor \(a_{\text{Gap},i}\) thus expresses the relationship of the HP gap-type area to the total gap-type area of the machine. It seems desirable to work this out, as with constant wrap angles, tooth pairings with different numbers of teeth can be compared. As the number of teeth increases, the weighting factor \(a_{\text{Gap},i}\) is reduced, which corresponds to a reduction in the gap priority of the individual gap types, and a consequent improvement in volumetric efficiency.

All these performance figures are basically suitable for a qualitative assessment of machines based on their gap areas and values, but not for a quantitative representation of the volumetric efficiency or the overall efficiency of the compressor. They serve as a first step in the relative geometrical assessment of different compressor designs (e.g. in a computer-aided optimization process).

4 APPLICATION IN PROFILE OPTIMIZATION

After implementing the performance codes defined above, it is necessary to examine their validity in computer-aided profile generation. For this purpose, an optimization strategy for screw rotor profiles using evolutionary approaches is employed [5]. The representation of the rotor flanks is carried out by means of a non-uniform rational basis spline (NURBS) curve [6]. The geometrical parameters of the reference machine are listed in Table 1.

Further the general requirements for the optimization process are a sample size of 30 rotors and a maximum of 100 000 optimization steps. The curve of the rotor profile of a tooth is represented by a polynomial degree of 3, and 12 control points. The intermesh conditions for profile generation follow the general gearing law. Because of the varying influences of the gap types on the general course of the process during the compression phase, the front rotor gaps on the low-pressure side are ignored during the generation of the performance figures. These only have a marginal influence on the volumetric efficiency of the compressor compared to the other gap types which are taken into account.

In order to check the validity of the figures, the optimization process deliberately began with a reference profile which diverged very considerably from a modern standard profile, see Fig. 3. This is reflected in the very large relative area of the blow hole. The task was to check whether the minimized performance figures in use would modify the profile generation towards modern rotor profiles, changing the relation between the gap areas to bring them in line with normal relations in a modern screw compressor. The optimized profiles can be seen in Fig. 3.

The assessment of the relevant data \(\Pi_1\) to \(\Pi_3,0,CM\) was carried out by evaluating the percentage change in the gap area in relation to the volumetric efficiency of the compressor, see Fig. 4. Compared with the reference machine, which has its gap characteristics set at 100 per cent, basic differences between the individual figures can be seen.

The optimization results show that the profile lengths are increased by up to 10 per cent in all cases, while the area of the blow hole is considerably reduced. Minimizing \(\Pi_1\) reduces the blow-hole area by c. 65 per cent, the second code by c. 70 per cent while the last code reduces it by up to 90 per cent compared to the reference machine. It is clear that the profile intermesh gap has opposing characteristics to the blow-hole area, because reducing the blow-hole area basically results in enlarging the intermesh clearance. This does not necessarily mean that there is a linear interconnection between these gaps, as code \(\Pi_2\) allows a greater change in the intermesh profile in relation to \(\Pi_3,0,CM\), resulting only in a smaller percentage gap change in the blow hole.

The operating code, which did not significantly reduce the area of the blow hole, shows larger changes in the casing gap instead. This can be explained by the increased crown circle diameter of the rotors. This results in a reduction in the area of the rotor front gap. Code \(\Pi_3,0,CM\) reduces the area component, particularly on the female rotor side, as a result of a reduction in the female crown circle and also of a narrower profile of the female rotor itself. Using code \(\Pi_3,0,CM\) a maximum reduction in the blow-hole area can only be achieved by configuring a narrow, pointed female rotor form.

The percentage area distribution for each of the gap types in the compressor is shown in Fig. 5. The profile form of the reference machine has a gap-type

<table>
<thead>
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<th>Table 1 Parameters of the reference machine (subscript 11)</th>
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<tbody>
<tr>
<td>Rotor length</td>
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<tr>
<td>Tooth relation (male–female rotor)</td>
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<tr>
<td>Wrap angle (male–female rotor)</td>
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<tr>
<td>Gap height setting</td>
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distribution which is untypical for dry-running screw compressors. This choice of profile has led to a very large blow-hole area, which takes up 58 per cent of the hole-gap area. The second-largest area is accounted for by the intermesh clearance, which accounts for 36 per cent of the gap area. The male and female rotor casing gaps each take up 3 per cent of the total area. The percentage for the front gap on the HP side is very small by comparison, amounting to less than 1 per cent.

Minimizing the operating values $\Pi_1$ to $\Pi_3,OCM$ leads in each case to a different gap area distribution. This gap distribution expresses the weighting of the individual gaps within the framework of the codes, dependent on the geometrical parameters. All the operational codes effect a reduction in the blow-hole area to different degrees, plus an increase in the profile intermesh area and in the housing gap. The role played by the front gap remains a minor one. A comparison of the codes reveals that code $\Pi_3,OCM$ achieves a maximum intermesh area of 77 per cent, an area ratio for the blow hole of 12 per cent, and 5 per cent each for the housing gaps.

A comparison with a dry-running standard rotor profile currently in use shows that the thrust of the optimization process, including the percentage ratios of the gap types, with a view to their application in the screw compressor field, is along the right lines, see Fig. 5. In particular, code $\Pi_3,OCM$ can achieve an area distribution of the gap types which meets modern standards. A possible reason for the imprecise implementation of the gap area relations can be put down to limitations in the profile generation process and adherence to the general gearing law, as modern standard profiles do not necessarily observe gearing criteria.
5 MODIFYING GEOMETRICAL PARAMETERS

Within the development framework for operational codes, the influence of geometrical changes on the thermodynamic operating behaviour needs to be evaluated. In view of the good results for profile optimization, the effect of changing wrap angle and gap height was also examined, employing code $\Pi_{3,OCM}$, see Fig. 6. This procedure was carried out using the optimization solution for code $\Pi_{3,OCM}$. The reference point was a wrap angle of $200^\circ$. The angle could be varied between $140^\circ$ and $220^\circ$.

It can be observed that when the wrap angle is small, the performance figure relation is $>1$. Figures $>1$ indicate a deteriorating effect on volumetric efficiency, since the chamber volume remains virtually unchanged because of the small wrap angle. Consequently, the only way to change the gap area ratios is to alter the wrap angle. However, with larger wrap angles, declining performance value relationship can be observed. Minimizing performance value relations should lead to an enhanced influence on volumetric efficiency, with the gradient becoming continuously less steep during the shift from small to large wrap angles. As the wrap angle increases, the chamber volume is gradually reduced, resulting in a larger-scale compressor. Within the code formation process, gap distribution outweighs an increasing wrap angle in comparison with an increase in machine size, but this indicates a decreasing influence on volumetric efficiency.

Furthermore, as the wrap angle was changed, the gap height of a single gap type was altered by 0.1 mm. The effect of an increase in gap type, in particular of the intermesh gap, on the volumetric efficiency of a screw compressor has been adequately researched [4]. The results obtained in reference [4] can, up to a certain point, be transferred to all dry-running screw superchargers with a tooth relationship of 3–5. This makes it clear that the intermesh and housing gaps are the principal influences on volumetric efficiency. As far as the housing gaps are concerned, the male casing gap influences performance considerably more than the female casing gap. This can be related to the low tooth count of the male rotor. In comparison to these gaps,
the HP-side front gap has little influence on volumetric efficiency.

This behaviour is also represented in operating code $\Pi_{3,OCM}$. With constant wrap angle and gap changes of 0.1 mm, altering the intermesh clearance has the greatest influence on the operating code relation. Next in line are the casing gap on the male rotor side, the casing gap on the female side, and a very minor influence from the front gap of the rotors on the HP side.

The degree of gap influence on changes in the operating code relation varies via the wrap angle area. In comparison with small wrap angles, the relative influence of individual gaps rises, with the influence of the intermesh clearance and the male rotor casing gap rising significantly. This can be explained in terms of the low tooth count of the male rotor compared with the female rotor. As the wrap angle rises, on the other hand, changes in the gap heights have a much smaller influence on the operating code, so that the influence of the low tooth count on the male rotor plays a smaller and smaller role. However, it generally remains the case that, with constant gap height variation, intermesh clearance has the greatest influence on the operating code and volumetric efficiency relations.

### 6 SUMMARY/FUTURE PROSPECTS

In this study operating codes for the comparative evaluation of the operating behaviour of screw compressors, derived purely from geometrical parameters for this type of machine, have been described. Compared with operating codes derived from a 2D representation, the codes introduced here, which take into account rotor length and wrap angle, achieve greater validity in the evaluation. Internal and external weighting factors, which connect and compare gap priorities both directly and interactively, make it possible to extend the validity of the operating codes. If the codes introduced here are deployed in connection with a profile optimization, the values related to the HP chamber emerge as goal orientated. In conclusion, the development of geometrical performance codes shows that it is possible to compare different rotor designs with one another by this means, and to evaluate them. The interrelationships between geometrical parameters described here provide the option of evaluating rotor profiles in terms of their comparative efficiency at an early stage of the development process.

The next step will be to examine the applicability of the codes to various other geometrical parameters such as the length–diameter ratio, and to deal with differing gap heights. Either experimentally measured data or comprehensive thermodynamic simulation calculations (e.g. via KaSim [3, 4]), can be used as a basis for comparison. It remains desirable to extend geometrical relations from pure rotor geometry to compressor geometry in general, so that the influence of charge cycle clearances during the energy conversion process can also be determined.

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### APPENDIX

#### Notation

- $A$: gap area (m$^2$)
- BH: blow hole
- CG: casing gap
- $e$: external
- $f$: delivered
- FG: front gap
- FR: female rotor
- $h$: height (m)
- HP: high pressure
- $i$: number, internal
- IMC: intermesh clearance
- $l$: length (m)
- LP: low pressure
- $m$: mass flow (kg/s)
- max: maximum
- MR: male rotor
- OCM: one-chamber Model
- $th$: theoretical
- Type: type of gap
- $V$: volume (m$^3$)
- $z$: number of lobes
- $l$: number, Gap number
- $i$: reference subscript
- $\alpha$: alpha (degree), factor
- $\lambda$: volumetric efficiency
- $\Pi$: performance code
- $\phi$: wrap angle (degree)